Studies of Angular Correlations in the Decays $B_s^0 \to J/\psi\,\phi$ by Using the SIMUB Generator

A. Bel'kov[1], S. Shulga[2]

[1] Particle Physics Laboratory, Joint Institute for Nuclear Research,
141980 Dubna, Moscow region, Russia
[2] Francisk Skarina Gomel State University, Gomel, Belarus

Abstract

The performance of the method of angular moments on the $\Delta\Gamma_s$ determination from analysis of untagged decays $B_s^0(t), \overline{B}_s^0(t) \to J/\psi(\to l^+l^-)\,\phi(\to K^+K^-)$ is examined by using the SIMUB generator. The results of Monte Carlo studies with evaluation of measurement errors are presented. The method of angular moments gives stable results for the estimate of $\Delta\Gamma_s$ and is found to be an efficient and flexible tool for the quantitative investigation of the $B_s^0 \to J/\psi\,\phi$ decay. The statistical error of the ratio $\Delta\Gamma_s/\Gamma_s$ for values of this ratio in the interval $[0.03, 0.3]$ was found to be independent on this value, being 0.015 for 10^5 events.

1   Introduction

The study of decays $B_s^0(t), \overline{B}_s^0(t) \to J/\psi(\to l^+l^-)\,\phi(\to K^+K^-)$, which is one of the gold plated channels for B-physics studies at the LHC, looks very interesting from the physics point of view. It presents several advantages related to the dynamics of these decays, characterized by proper-time-dependent angular distributions, which can be described in terms of bilinear combinations of transversity amplitudes. Their time evolution involves, besides the values of two transversity amplitudes at the proper time $t = 0$ and their relative strong phases, the following fundamental parameters: the difference and average value of decay rates of heavy and light mass eigenstates of B_s^0 meson, $\Delta\Gamma_s$ and Γ_s, respectively, their mass difference ΔM_s, and the CP-violating weak phase $\phi_c^{(s)}$. The angular analysis of the decays $B_s^0(t), \overline{B}_s^0(t) \to J/\psi(\to l^+l^-)\,\phi(\to K^+K^-)$ provides complete determination of the transversity amplitudes and, in principle, gives the access to all these parameters.

In the present paper we examine the performance of the angular-moments method [1] applied to the angular analysis of untagged decays $B_s^0(t), \overline{B}_s^0(t) \to J/\psi(\to l^+l^-)\,\phi(\to K^+K^-)$ for the determination of $\Delta\Gamma_s$. After giving the physics motivation in Section 2, we describe in the next section the method of angular moments based on weighting functions introduced in Ref. [1]. For the case of $\Delta\Gamma_s$ determination this method is properly modified in Section 4. The SIMUB-package [2] for physics simulation of B-meson production and decays has been used for Monte Carlo studies. A general information about the SIMUB package one can find in Appendix. The dynamics of the decays $B_s^0(t), \overline{B}_s^0(t) \to J/\psi(\to l^+l^-)\,\phi(\to K^+K^-)$ is described by four-dimensional probability density functions depending on decay time and three physical angles. The algorithms of multidimensional random number generation have been elaborated and then implemented in the package SIMUB to provide tools for Monte Carlo simulation of sequential two-body decays $B_s^0(t), \overline{B}_s^0(t) \to J/\psi(\to l^+l^-)\,\phi(\to K^+K^-)$ in accordance with theoretical time-dependent angular distributions. This algorithms is discussed in Section 5. In Section 6 we present the results of the Monte Carlo stud-

ies of the angular analysis of the decays $B_s^0(t), \overline{B}_s^0(t) \to J/\psi(\to l^+l^-)\,\phi(\to K^+K^-)$ with two sets of weighting functions for angular-moments method and concentrate on the evaluation of measurement errors and their dependence on statistics.

2   Phenomenological description of the decays $B_s^0(t), \overline{B}_s^0(t) \to J/\psi(\to l^+l^-)\,\phi(\to K^+K^-)$

The angular distributions for decays $B_s^0(t), \overline{B}_s^0(t) \to J/\psi(\to l^+l^-)\,\phi(\to K^+K^-)$ are governed by spin-angular correlations (see [3]-[6]) and involve three physically determined angles. In case of the so-called helicity frame [5], which is used in the present paper, these angles are defined as follows (see Fig. 1):

- The z-axis is defined to be the direction of ϕ-particle in the rest frame of the B_s^0. The x-axis is defined as any arbitrary fixed direction in the plane normal to the z-axis. The y-axis is then fixed uniquely via $y = z \times x$ (right-handed coordinate system).

- The angles $(\Theta_{l^+}, \chi_{l^+})$ specify the direction of the l^+ in the J/ψ rest frame while $(\Theta_{K^+}, \chi_{K^+})$ give the direction of K^+ in the ϕ rest frame. Since the orientation of the x-axis is a matter of convention, only the difference $\chi = \chi_{l^+} - \chi_{K^+}$ of the two azimuthal angles is physically meaningful.

In the most general form the angular distribution for the decay $B_s^0(t) \to J/\psi(\to l^+l^-)\,\phi(\to K^+K^-)$ in case of a *tagged B_s^0 sample* can be expressed as

$$\frac{d^4N^{tag}(B_s^0)}{d\cos\Theta_{l^+}\,d\cos\Theta_{K^+}\,d\chi\,dt} = \frac{9}{32\pi}\sum_{i=1}^{6}\mathcal{O}_i(t)g_i(\Theta_{l^+}, \Theta_{K^+}, \chi)\,. \tag{1}$$

Here $\mathcal{O}_i$ $(i = 1, ..., 6)$ are time-dependent bilinear combinations of the transversity amplitudes $A_0(t)$, $A_{||}(t)$ and $A_\perp(t)$ for the weak transition $B_s^0(t) \to J/\psi\,\phi$ [7] (we treat these combinations as observables):

$$\mathcal{O}_1 = |A_0(t)|^2\,, \quad \mathcal{O}_2 = |A_{||}(t)|^2\,, \quad \mathcal{O}_3 = |A_\perp(t)|^2\,,$$

$$\mathcal{O}_4 = \text{Im}\left(A_{||}^*(t)A_\perp(t)\right), \quad \mathcal{O}_5 = \text{Re}\left(A_0^*(t)A_{||}(t)\right), \quad \mathcal{O}_6 = \text{Im}\left(A_0^*(t)A_\perp(t)\right), (2)$$

and the g_i are functions of the angles Θ_{l^+}, Θ_{K^+}, χ only [5]:

$$g_1 = 2\cos^2\Theta_{K^+}\sin^2\Theta_{l^+},$$

$$g_2 = \sin^2\Theta_{K^+}(1 - \sin^2\Theta_{l^+}\cos^2\chi),$$

$$g_3 = \sin^2\Theta_{K^+}(1 - \sin^2\Theta_{l^+}\sin^2\chi),$$

$$g_4 = -\sin^2\Theta_{K^+}\sin^2\Theta_{l^+}\sin 2\chi,$$

$$g_5 = \frac{1}{\sqrt{2}}\sin 2\Theta_{l^+}\sin 2\Theta_{K^+}\cos\chi,$$

$$g_6 = \frac{1}{\sqrt{2}}\sin 2\Theta_{l^+}\sin 2\Theta_{K^+}\sin\chi. \tag{3}$$

For the decay $\overline{B}_s^0(t) \to J/\psi(\to l^+l^-)\,\phi(\to K^+K^-)$ in case of a *tagged* $\overline{B}_s^0$ *sample* the angular distribution is given by

$$\frac{d^4N^{tag}(\overline{B}_s^0)}{d\cos\Theta_{l^+}d\cos\Theta_{K^+}d\chi dt} = \frac{9}{32\pi}\sum_{i=1}^{6}\overline{\mathcal{O}}_i(t)g_i(\Theta_{l^+}, \Theta_{K^+}, \chi) \tag{4}$$

with the same angular functions g_i and

$$\overline{\mathcal{O}}_1 = |\bar{A}(t)|^2, \quad \overline{\mathcal{O}}_2 = |\bar{A}_{||}(t)|^2, \quad \overline{\mathcal{O}}_3 = |\bar{A}_\perp(t)|^2,$$

$$\overline{\mathcal{O}}_4 = \text{Im}\left(\bar{A}_{||}^*(t)\bar{A}_\perp(t)\right), \quad \overline{\mathcal{O}}_5 = \text{Re}\left(\bar{A}_0^*(t)\bar{A}_{||}(t)\right), \quad \overline{\mathcal{O}}_6 = \text{Im}\left(\bar{A}_0^*(t)\bar{A}_\perp(t)\right), (5)$$

where $\bar{A}_0(t)$, $\bar{A}_{||}(t)$ and $\bar{A}_\perp(t)$ are the transversity amplitudes for the transition $\overline{B}_s^0(t) \to J/\psi\,\phi$.

The time dependence of the transversity amplitudes for the transitions $B_s^0(t), \overline{B}_s^0(t) \to J/\psi\,\phi$ is not of purely exponential form due to the presence of $B_s^0 - \overline{B}_s^0$ mixing. This mixing arises due to either a mass difference or a decay-width difference between the mass eigenstates of the $(B_s^0 - \overline{B}_s^0)$ system. The time evolution of the state $|B_s^0(t)\rangle$ of an initially, i.e. at time $t = 0$, present B_s^0 meson can be described in general form as follows:

$$|B_s^0(t)\rangle = g_+(t)|B_s^0\rangle + g_-(t)|\overline{B}_s^0\rangle, \qquad g_+(t = 0) = 1, \quad g_-(t = 0) = 0,$$

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i.e., the state $|B_s^0(t)\rangle$ at time t is a mixture of the flavor states $|B_s^0\rangle$ and $|\overline{B}_s^0\rangle$ with probabilities defined by the functions $g_+(t)$ and $g_-(t)$. In analogous way, the time evolution of the state $|\overline{B}_s^0(t)\rangle$ of an initially present $\overline{B}_s^0$ meson is described by the relation

$$|\overline{B}_s^0(t)\rangle = \bar{g}_+(t)|B_s^0\rangle + \bar{g}_-(t)|\overline{B}_s^0\rangle\,, \qquad \bar{g}_+(t=0) = 0\,, \quad \bar{g}_-(t=0) = 1\,.$$

Diagonalization of the full Hamiltonian (see [8] for more details) gives

$$g_+(t) = \frac{1}{2}\left(e^{-i\mu_L t} + e^{-i\mu_H t}\right)\,, \quad g_-(t) = \frac{\alpha}{2}\left(e^{-i\mu_L t} - e^{-i\mu_H t}\right)\,,$$
$$\bar{g}_+(t) = g_-(t)/\alpha^2\,, \qquad\qquad \bar{g}_-(t) = g_+(t)\,. \tag{6}$$

Here $\mu_{L/H} \equiv M_{L/H} - (i/2)\Gamma_{L/H}$ are eigenvalues of the full Hamiltonian corresponding to the masses and total widths of "light" and "heavy" eigenstates $|B_{L/H}\rangle$, and α is a phase factor defining the CP transformation of flavor eigenstates of the neutral B_s-meson system: $CP|B_s^0\rangle = \alpha|\overline{B}_s^0\rangle$. In the case $|\alpha| \neq 1$ the probability for B_s^0 to oscillate to a $\overline{B}_s^0$ is not equal to the probability of a $\overline{B}_s^0$ to oscillate to a B_s^0. Such an asymmetry in mixing is often referred to as *indirect CP* violation, which is negligibly small in case of the neutral B-meson system.

The time evolution of the transversity amplitudes $A_f(t)$ ($f = 0, ||, \perp$) is given by the equations

$$A_f(t) = A_f(0)\left[g_+(t) + g_-(t)\frac{1}{\eta_{CP}^f\alpha}\xi_f^{(s)}\right]\,, \quad \bar{A}_f(t) = A_f(0)\left[\bar{g}_+(t) + \bar{g}_-(t)\frac{1}{\eta_{CP}^f\alpha}\xi_f^{(s)}\right]\,. \tag{7}$$

Here η_{CP}^f are eigenvalues of CP-operator acting on the transversity components of the final state which are eigenstates of CP-operator:

$$CP|J/\psi\,\phi\rangle_f = \eta_{CP}^f|J/\psi\,\phi\rangle_f\,, \quad (f = 0, ||, \perp)\,,$$
$$\eta_{CP}^0 = 1\,, \quad \eta_{CP}^{||} = 1\,, \quad \eta_{CP}^\perp = -1\,,$$

and $\xi_f^{(s)}$ is the CP-violating weak phase [9]:

$$\xi_f^{(s)} = e^{-i\phi_c^{(s)}}\,, \quad \phi_c^{(s)} = 2[\arg(V_{ts}^*V_{tb}) - \arg(V_{cq}^*V_{cb})] = -2\delta\gamma\,,$$

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where δ is the complex phase in the standard parameterization of the CKM matrix elements V_{ij} ($i \in \{u, c, t\}$, $j \in \{d, s, b\}$), and γ is the third angle of the unitarity triangle.

The phase $\phi_c^{(s)}$ is very small and vanishes at leading order in the Wolfenstein expansion. Taking into account higher-order terms in the Wolfenstein parameter $\lambda = \sin\theta_C = 0.22$ gives a non-vanishing result [10]:

$$\phi_c^{(s)} = -2\lambda^2 \eta = -2\lambda^2 R_b \sin\gamma \, .$$

Here

$$R_b \equiv \frac{1}{\lambda} \frac{|V_{ub}|}{|V_{cb}|}$$

is constrained by present experimental data as $R_b = 0.36 \pm 0.08$ [11]. Using the estimate $\gamma = (59 \pm 13)^o$ [12], the following constrain can be obtained for the phase $\phi_c^{(s)}$:

$$\phi_c^{(s)} = -0.03 \pm 0.01 \, . \tag{8}$$

According to Eq. (7) at time $t = 0$, the transversity amplitudes of $B_s^0, \overline{B^0}_s \rightarrow J/\psi\,\phi$ decays depend on the same observables $|A_0(0)|$, $|A_{||}(0)|$, $|A_\perp(0)|$ and on the two CP-conserving strong phases, $\delta_1 \equiv arg[A_{||}^*(0)A_\perp(0)]$ and $\delta_2 \equiv arg[A_0^*(0)A_\perp(0)]$. Time-reversal invariance of strong interactions forces the form factors parameterizing quark currents to be all relatively real and, consequently, naive factorization leads to the following common properties of the observables:

$$\text{Im}[A_0^*(0)A_\perp(0)] = 0 \, , \quad \text{Im}[A_{||}^*(0)A_\perp(0)] = 0 \, , \quad \text{Re}[A_0^*(0)A_{||}(0)] = \pm|A_0(0)A_{||}(0)| \, .$$

Moreover, in the absence of strong final-state interactions, $\delta_1 = \pi$ and $\delta_2 = 0$.

In the framework of the effective Hamiltonian approach the two body decays, both $B_s^0 \rightarrow J/\psi\,\phi$ and $B_d^0 \rightarrow J/\psi\,K^\star$, correspond to the transitions $\bar{b} \rightarrow \bar{s}c\bar{c}$ with topologies of color-suppressed spectator diagrams shown in Fig. 2. Factorizing the hadronic matrix elements of the four-quark operators of the effective Hamiltonian into hadronic matrix elements of quark currents, the transversity amplitudes $|A_0(0)|$,

$|A_{||}(0)|$, $|A_\perp(0)|$ of decays $B_q^0, \overline{B}_q^0 \to J/\psi V((q, V) \in \{(s, \phi), (d, K^\star)\})$ can be expressed in terms of effective Wilson coefficient functions, constants of J/ψ decay, and form factors of transitions $B_q \to V$ induced by quark currents [1]. In Table 1 we collect the predictions of Ref. [1] for the transversity amplitudes of $B_s^0 \to J/\psi\,\phi$ ($B_d^0 \to J/\psi K^\star$) calculated with $B \to K^\star$ form factors given by different models [13, 14, 15]. The $B \to K^\star$ form factors can be related to the $B \to \phi$ case by using SU(3) flavor symmetry. The most precise polarization measurements performed recently in decays $B \to J/\psi\, K^\star$:

$$|A_0(0)|^2 = 0.60 \pm 0.04\,, \quad |A_\perp(0)|^2 = 0.16 \pm 0.03 \quad (\text{BaBar } [16])\,,$$
$$|A_0(0)|^2 = 0.62 \pm 0.04\,, \quad |A_\perp(0)|^2 = 0.19 \pm 0.04 \quad (\text{Belle } [17])\,,$$

confirm the predictions based on the model [15].

3   Angular-moments method

The angular distributions for decays $B_s^0(t), \overline{B}_s^0(t) \to J/\psi(\to l^+l^-)\,\phi(\to K^+K^-)$ in case of tagged B_s^0 and $\overline{B}_s^0(t)$ samples (see Eqs. (1) and (4), respectively) as well as in case of the untagged sample can be expressed in the most general form in terms of observables $b_i(t)$:

$$f_0(\Theta_{l^+}, \Theta_{K^+}, \chi; t) = \frac{9}{32\pi} \sum_{i=1}^{6} b_i(t) g_i(\Theta_{l^+}, \Theta_{K^+}, \chi)\,. \tag{9}$$

The explicit time dependence of observables is given by the following relations:

$$b_1(t) = |A_0(0)|^2 G_L(t)\,,$$
$$b_2(t) = |A_{||}(0)|^2 G_L(t)\,,$$
$$b_3(t) = |A_\perp(0)|^2 G_H(t)\,,$$
$$b_4(t) = |A_{||}(0)|\,|A_\perp(0)|\,Z_1(t)\,,$$
$$b_5(t) = |A_0(0)|\,|A_{||}(0)|\,G_L(t)\cos(\delta_2 - \delta_1)\,,$$
$$b_6(t) = |A_0(0)|\,|A_\perp(0)|\,Z_2(t)\,, \tag{10}$$

where we have used the general compact notations:

$$G_{L/H}(t) = \frac{1}{2}\left[(1 \pm \cos\phi_c^{(s)})e^{-\Gamma_L t} + (1 \mp \cos\phi_c^{(s)})e^{-\Gamma_H t}\right],$$

$$Z_{1,2}(t) = \frac{1}{2}\left(e^{-\Gamma_H t} - e^{-\Gamma_L t}\right)\cos\delta_{1,2}\sin\phi_c^{(s)}$$

– for observables $b_i \equiv (\mathcal{O}_i + \overline{\mathcal{O}}_i)/2$ in case of the untagged sample with equal initial numbers of B_s^0 and $\overline{B}_s^0$, while

$$G_{L/H}^{(B_s^0)/(\overline{B}_s^0)}(t) = G_{L/H}(t) \pm e^{-\Gamma_s t}\sin(\Delta Mt)\sin\phi_c^{(s)},$$

$$Z_{1,2}^{(B_s^0)/(\overline{B}_s^0)}(t) = Z_{1,2}(t) \pm e^{-\Gamma_s t}\left[\sin\delta_{1,2}\cos(\Delta Mt) - \cos\delta_{1,2}\sin(\Delta Mt)\sin\phi_c^{(s)}\right]$$

– for observables $b_i^{(B_s^0)} \equiv \mathcal{O}_i$ and $b_i^{(\overline{B}_s^0)} \equiv \overline{\mathcal{O}}_i$ in case of tagged B_s^0 and $\overline{B}_s^0(t)$ samples, respectively, with $\Gamma_s \equiv (\Gamma_L + \Gamma_H)/2$. It is easy to see that both in the tagged and untagged case we have

$$G_{L/H}(t)\big|_{\phi_c^{(s)}=0} = e^{-\Gamma_{L/H} t}.$$

According to Ref. [1], the observables $b_i(t)$ can be extracted from distribution function (9) by means of weighting functions $w_i(\Theta_{l^+}, \Theta_{K^+}, \chi)$ for each i such that

$$\frac{9}{32\pi}\int d\cos\Theta_{l^+}d\cos\Theta_{K^+}d\chi\ w_i(\Theta_{l^+}, \Theta_{K^+}, \chi)\ g_j(\Theta_{l^+}, \Theta_{K^+}, \chi) = \delta_{ij}, \qquad (11)$$

projecting out the desired observable alone:

$$b_i(t) = \int d\cos\Theta_{l^+}d\cos\Theta_{K^+}d\chi\ f_0(\Theta_{l^+}, \Theta_{K^+}, \chi\,;t)\ w_i(\Theta_{l^+}, \Theta_{K^+}, \chi). \qquad (12)$$

The angular-distribution function (9) obeys the condition

$$L(t) \equiv \int d\cos\Theta_{l^+}d\cos\Theta_{K^+}d\chi\ f_0(\Theta_{l^+}, \Theta_{K^+}, \chi\,;t)\ = b_1(t) + b_2(t) + b_3(t). \qquad (13)$$

For decays $B \to J/\psi(\to l^+l^-)\,\phi(\to K^+K^-)$, the explicit expressions of weighting functions, given in Table 5 of Ref. [1] for physically meaningful angles in the transversity frame, get the following form (Set A) after transformation into the helicity frame:

$$w_1^{(A)} = 2 - 5\cos^2\Theta_{l^+},$$

$$w_2^{(A)} = 2 - 5 \sin^2 \Theta_{l+} \cos^2 \chi \,,$$

$$w_3^{(A)} = 2 - 5 \sin^2 \Theta_{l+} \sin^2 \chi \,,$$

$$w_4^{(A)} = -\frac{5}{2} \sin^2 \Theta_{K+} \sin 2\chi \,,$$

$$w_5^{(A)} = \frac{25}{4\sqrt{2}} \sin 2\Theta_{K+} \sin 2\Theta_{l+} \cos \chi \,,$$

$$w_6^{(A)} = \frac{25}{4\sqrt{2}} \sin 2\Theta_{K+} \sin 2\Theta_{l+} \sin \chi \,. \tag{14}$$

The expressions of Eq. (14) are not unique and there are many legitimate choices of weighting functions. A particular set can be derived by linear combination of angular functions g_i (see [1] for more discussions):

$$w_i(\Theta_{l+}, \Theta_{K+}, \chi) = \sum_{j=1}^{6} \lambda_{ij} g_j(\Theta_{l+}, \Theta_{K+}, \chi) \,, \tag{15}$$

where the 36 unknown coefficients λ_{ij} are solutions of 36 equations

$$\frac{9}{32\pi} \sum_{j=1}^{6} \lambda_{ij} \int d\cos\Theta_{l+} \, d\cos\Theta_{K+} \, d\chi \; g_j(\Theta_{l+}, \Theta_{K+}, \chi) \, g_k(\Theta_{l+}, \Theta_{K+}, \chi) = \delta_{ik} \,. \tag{16}$$

The weighting functions (set B) corresponding to the linear combination of the angular functions (3) are given by

$$
\begin{aligned}
w_1^{(B)} &= \frac{1}{12}[28 \cos^2 \Theta_{K+} \sin^2 \Theta_{l+} - 3 \sin^2 \Theta_{K+}(1 + \cos^2 \Theta_{l+})] \,, \\
w_2^{(B)} &= -\frac{1}{8}[4 \cos^2 \Theta_{K+} \sin^2 \Theta_{l+} - 29 \sin^2 \Theta_{K+}(1 - \sin^2 \Theta_{l+} \cos^2 \chi) \\
&\quad + 21 \sin^2 \Theta_{K+}(1 - \sin^2 \Theta_{l+} \sin^2 \chi)] \,, \\
w_3^{(B)} &= -\frac{1}{8}[4 \cos^2 \Theta_{K+} \sin^2 \Theta_{l+} + 21 \sin^2 \Theta_{K+}(1 - \sin^2 \Theta_{l+} \cos^2 \chi) \\
&\quad - 29 \sin^2 \Theta_{K+}(1 - \sin^2 \Theta_{l+} \sin^2 \chi)] \,, \\
w_4^{(B)} &= -\frac{25}{8} \sin^2 \Theta_{K+} \sin^2 \Theta_{l+} \sin 2\chi \,, \\
w_5^{(B)} &= w_5^{(A)} \,, \\
w_6^{(B)} &= w_6^{(A)} \,. \tag{17}
\end{aligned}
$$

For a limited number of experimental events N in the time bin around the fixed value of the proper time t, distributed according to the angular function (9), it is

convenient to introduce the normalized observables

$$\bar{b}_i(t) \equiv b_i(t)/L(t) \tag{18}$$

with normalization factor $L(t)$ given by Eq. (13). Then, as it follows from the Eq. (12), the observables $\bar{b}_i(t)$ (18) are measured experimentally by

$$\bar{b}_i^{(exp)} = \frac{1}{N} \sum_{j=1}^{N} w_i^j \tag{19}$$

with summation over events in a time bin around t. Here $w_i^j \equiv w_i(\Theta_{l+}^j, \Theta_{K+}^j, \chi^j)$, where Θ_{l+}^j, Θ_{K+}^j and χ^j are angles measured in the j-th event. The statistical measurement error of the observable (19) can be estimated as

$$\delta\bar{b}_i^{(exp)} = \frac{1}{N} \sqrt{\sum_{j=1}^{N} (\bar{b}_i^{(exp)} - w_i^j)^2} \,,$$

with summation over all events in the same time bin.

4 Time-integrated observables

For data analysis it is rather convenient to use the time-integrated observables [18] defined as

$$\tilde{b}_i(T_0) = \frac{1}{\tilde{L}(T)} \int_0^{T_0} dt \int d\cos\Theta_{l+} \, d\cos\Theta_{K+} \, d\chi \, w_i(\Theta_{l+}, \Theta_{K+}, \chi) \, f_0(\Theta_{l+}, \Theta_{K+}, \chi; t) \tag{20}$$

with argument $T_0 \leq T$, where T is the maximal value of the B-meson proper time measured for the sample of events being used, and $\tilde{L}(T)$ is a new normalization factor, which has the form:

$$\begin{aligned}
\tilde{L}(T) &\equiv \int_0^T L(t) = \int_0^T dt \int d\cos\Theta_{l+} \, d\cos\Theta_{K+} \, d\chi \, f_0(\cos\Theta_{l+}, \cos\Theta_{K+}, \chi; t) = \\
&= (|A_0(0)|^2 + |A_{||}(0)|^2) \, \tilde{G}_L(T) + |A_\perp(0)|^2 \, \tilde{G}_H(T) \,,
\end{aligned} \tag{21}$$

where, in the compact notations used in Eq. (10),

$$\tilde{G}_{L/H}(T) \equiv \int_0^T dt \, G_{L/H}(t) \,.$$

The following normalization condition is valid for the observables (20): $\tilde{b}_1 + \tilde{b}_2 + \tilde{b}_3 = 1$. For a limited number of experimental events $N(T)$, measured in the proper time region $t \in [0, T]$, Eq. (20) reduces to

$$\tilde{b}_i^{(exp)}(T_0) = \frac{1}{N(T)} \sum_{j=1}^{N(T_0)} w_i^j \tag{22}$$

with summation over all events $N(T_0)$ in the time interval $t \in [0, T_0]$ for $T_0 \leq T$. In case of the untagged sample we have

$$\tilde{G}_{L/H}(T) = -\frac{1}{2} \left[(1 \pm \cos\phi_c^{(s)}) \frac{e^{-\Gamma_L T} - 1}{\Gamma_L} + (1 \mp \cos\phi_c^{(s)}) \frac{e^{-\Gamma_H T} - 1}{\Gamma_H} \right] \tag{23}$$

and

$$\begin{aligned} \tilde{Z}(T) &\equiv \frac{1}{\cos\delta_{1,2} \sin\phi_c^{(s)}} \int_0^T dt\, Z_{1,2}(T) \\ &= -\frac{1}{2} \left[(e^{-\Gamma_H T} - 1)/\Gamma_H - (e^{-\Gamma_L T} - 1)/\Gamma_L \right]. \end{aligned}$$

For the untagged sample the explicit form of time-integrated normalized observables (20) in terms of the functions $\tilde{G}_{L/H}(T)$ and $\tilde{Z}(T)$ is given by

$$\begin{aligned} \tilde{b}_1(T_0) &= |A_0(0)|^2 \, \tilde{G}_L(T_0)/\tilde{L}(T) \,, \\ \tilde{b}_2(T_0) &= |A_{||}(0)|^2 \, \tilde{G}_L(T_0)/\tilde{L}(T) \,, \\ \tilde{b}_3(T_0) &= |A_\perp(0)|^2 \, \tilde{G}_H(T_0)/\tilde{L}(T) \,, \\ \tilde{b}_4(T_0) &= |A_{||}(0)|\, |A_\perp(0)|\, \tilde{Z}(T_0) \cos\delta_1 \sin\phi_c^{(s)}/\tilde{L}(T) \,, \\ \tilde{b}_5(T_0) &= |A_0(0)|\, |A_{||}(0)|\, \tilde{G}_L(T_0) \cos(\delta_2 - \delta_1)/\tilde{L}(T) \,, \\ \tilde{b}_6(T_0) &= |A_0(0)|\, |A_\perp(0)|\, \tilde{Z}(T_0) \cos\delta_2 \sin\phi_c^{(s)}/\tilde{L}(T) \,. \end{aligned} \tag{24}$$

In the Standard Model (SM) $\sin\phi_c^{(s)} \approx 0$ and the observables $\tilde{b}_{4,6}(T_0)$ are vanishing. In case of a new physics signal the values of $\sin\phi_c^{(s)}$ and $\tilde{b}_{4,6}(T_0)$ can be sizable, however.

The following relations are valid for the observables (24):

$$\tilde{b}_4(T_0) = \cos\delta_1 \sin\phi_c^{(s)}\, \tilde{Z}(T_0) \sqrt{\frac{\tilde{b}_2(T_0)\, \tilde{b}_3(T_0)}{\tilde{G}_L(T_0)\, \tilde{G}_H(T_0)}} \,,$$

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$$\tilde{b}_5(T_0) = \cos(\delta_2 - \delta_1) \sqrt{\tilde{b}_1(T_0)\, \tilde{b}_2(T_0)}\,,$$

$$\tilde{b}_6(T_0) = \cos\delta_2 \sin\phi_c^{(s)}\, \tilde{Z}(T_0) \sqrt{\frac{\tilde{b}_1(T_0)\, \tilde{b}_3(T_0)}{\tilde{G}_L(T_0)\, \tilde{G}_H(T_0)}}\,.$$

If we introduce the function

$$\tilde{\gamma}(T) \equiv \tilde{G}_H(T)/\tilde{G}_L(T)\,, \tag{25}$$

then, the values of initial transversity amplitudes at $t = 0$ and the strong-phase difference $(\delta_2 - \delta_1)$ are determined from the observables $\tilde{b}_i(T) \equiv \tilde{b}_i(T = T_0)$ by

$$|A_0(0)|^2 = \frac{\tilde{b}_1(T)}{\tilde{b}_1(T) + \tilde{b}_2(T) + \tilde{b}_3(T)/\tilde{\gamma}(T)}\,,$$

$$|A_\|(0)|^2 = \frac{\tilde{b}_2(T)}{\tilde{b}_1(T) + \tilde{b}_2(T) + \tilde{b}_3(T)/\tilde{\gamma}(T)}\,,$$

$$|A_\perp(0)|^2 = \frac{\tilde{b}_3(T)/\tilde{\gamma}(T)}{\tilde{b}_1(T) + \tilde{b}_2(T) + \tilde{b}_3(T)/\tilde{\gamma}(T)}\,,$$

$$\cos(\delta_2 - \delta_1) = \frac{\tilde{b}_5(T)}{\sqrt{\tilde{b}_1(T)\, \tilde{b}_2(T)}}\,, \tag{26}$$

where we consider the initial amplitudes normalized as $|A_0(0)|^2 + |A_\|(0)|^2 + |A_\perp(0)|^2 = 1$. We have also:

$$\sin\phi_c^{(s)} \cos\delta_{1,2} = \frac{\tilde{b}_{4,6}(T)}{\sqrt{\tilde{b}_{2,1}(T)\tilde{b}_3(T)}} \frac{\sqrt{\tilde{G}_L(T)\, \tilde{G}_H(T)}}{\tilde{Z}(T)}\,. \tag{27}$$

For extraction of the B_s^0-width difference $\Delta\Gamma_s \equiv \Gamma_H - \Gamma_L$ from experimental data it is convenient to use a special set of the time-integrated normalized observables [18]:

$$\hat{b}_i(T_0) = \frac{1}{\tilde{L}(T)} \int_0^{T_0} dt \int d\cos\Theta_{l^+} d\cos\Theta_{K^+} d\chi\, w_i(\Theta_{l^+}, \Theta_{K^+}, \chi)\, e^{\Gamma' t}\, f_0(\Theta_{l^+}, \Theta_{K^+}, \chi; t)\,, \tag{28}$$

where Γ' is some arbitrary initial approximation of the B_s^0-meson total decay width. These observables can be extracted from the experimental events $N(T)$, measured in the proper time region $t \in [0, T]$, by using the formula

$$\hat{b}_i^{(exp)}(T_0) = \frac{1}{N(T)} \sum_{j=1}^{N(T_0)} W_i^j\,, \tag{29}$$

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where $W_i^j \equiv e^{\Gamma' t^j} w_i^j$, and summation is performed over all events $N(T_0)$ in the time interval $t^j \in [0, T_0]$.

For the untagged sample, the explicit expressions for the time-integrated observables (28) can be easily obtained by replacing $\tilde{b}_i$, $\tilde{G}_{L/H}$ and $\tilde{Z}$ in the expressions of Eq. (24) by $\hat{b}_i$, $\hat{G}_{L/H}$ and $\hat{Z}$, respectively, (with the same normalization factor (21)) after introducing the following notations

$$
\begin{aligned}
\hat{G}_{L/H}(T) &\equiv \int_0^T dt\, e^{\Gamma' t}\, G_{L/H}(t) \\
&= (1 \pm \cos\phi_c^{(s)}) \frac{e^{\Delta\Gamma_L T/2} - 1}{\Delta\Gamma_L} - (1 \mp \cos\phi_c^{(s)}) \frac{e^{-\Delta\Gamma_H T/2} - 1}{\Delta\Gamma_H}, \quad (30) \\
\hat{Z}(T) &\equiv \frac{1}{\cos\delta_{1,2}\sin\phi_c^{(s)}} \int_0^T dt\, e^{\Gamma' t}\, Z_{1,2}(T) \\
&= \frac{1 - e^{\Delta\Gamma_L T/2}}{\Delta\Gamma_L} + \frac{1 - e^{-\Delta\Gamma_H T/2}}{\Delta\Gamma_H}.
\end{aligned}
$$

where $\Delta\Gamma_{L/H}$ are auxiliary parameters given by

$$
\Delta\Gamma_L = 2(\Gamma' - \Gamma_L), \qquad \Delta\Gamma_H = -2(\Gamma' - \Gamma_H). \quad (31)
$$

Eq. (26) is also valid after such a replacement.

5 Algorithms of multidimensional random number generation

The time-dependent angular distribution function (9) is used in the SIMUB generator [2] (see also Appendix) as density of the probability function for Monte Carlo simulation of the vertex and kinematics of the final-state particles in case of decays $B_s^0(t), \overline{B}_s^0(t) \rightarrow J/\psi(\rightarrow l^+ l^-)\,\phi(\rightarrow K^+ K^-)$. The variables of the function $f_0(\Theta_{l^+}, \Theta_{K^+}, \chi; t)$ can not be factorized and randomly generated in an independent way. Nevertheless, the random generation of $\cos\Theta_{l^+}$, $\cos\Theta_{K^+}$, χ and t can be performed either simultaneously according to the distribution function (9) by using four-dimensional random generator or successively, one after another, by us-

ing single-dimensional random number generators with accordance to distribution functions obtained by successive integration of the function (9) over its variables.

Let as consider the latter approach in the case of sequential random generation of the variables t, χ, $\cos\Theta_{l+}$ and $\cos\Theta_{K+}$. The following three distribution functions are used in this case:

$$f_1(\Theta_{l+},\chi;t) \equiv \int_{-1}^{+1} d\cos\Theta_{K+}\, f_0(\Theta_{l+},\Theta_{K+},\chi;t)\,,$$

$$f_2(\chi;t) \equiv \int_{-1}^{+1} d\cos\Theta_{l+}\, f_1(\Theta_{l+},\chi;t)\,, \tag{32}$$

and

$$f_3(t) \equiv \int_0^{2\pi} d\chi f_2(\chi;t) \;=\; b_1(t) + b_2(t) + b_3(t)\,. \tag{33}$$

The functions of Eqs. (32) and (33) present the most important experimentally observable distributions. In case of sequential decays $B_s^0(t), \overline{B}_s^0(t) \to J/\psi(\to l^+l^-)\,\phi(\to K^+K^-)$ the explicit form of the distribution functions (32) are given by

$$
\begin{aligned}
f_1(\cos\Theta_{l+},\chi;t) \;=\; & \frac{4}{3}b_1(t)\sin^2\Theta_{l+} + \frac{4}{3}b_2(t)\,(1-\sin^2\Theta_{l+}\cos^2\chi) \\
& +\frac{4}{3}b_3(t)\,(1-\sin^2\Theta_{l+}\sin^2\chi) - \mathcal{O}_4(t)\sin^2\Theta_{l+}\sin 2\chi\,, \\
f_2(\chi,t) \;=\; & b_1(t) + \frac{1}{2}b_2(t)\,(3-2\cos^2\chi) \\
& +\frac{1}{2}b_3(t)\,(3-2\sin^2\chi) - b_4(t)\sin 2\chi\,.
\end{aligned}
\tag{34}
$$

The procedure of random generation of the variables t, χ, $\cos\Theta_{l+}$ and $\cos\Theta_{K+}$ is as follows:

- First, the proper time t is randomly generated according to the distribution function $f_3(t)$.

- Second, the angle χ is randomly generated according to the single-dimensional distribution given by function $f_2(\chi;t)$ with t being fixed to be equal to the time, generated at the first step.

- Then, the value $\cos\Theta_{l+}$ is generated according to distribution function $f_1(\Theta_{l+},\chi;t)$ with values of t and χ fixed to be equal to their values generated at the previous two steps.

- Finally, the value of $\cos\Theta_{K+}$ is generated according to the single-dimensional distribution given by function $f_0(\Theta_{l+}, \Theta_{K+}, \chi; t)$ with a properly fixed values of t, χ and $\cos\Theta_{l+}$.

Two Monte Carlo methods of random numbers generation are implemented in the C++ class `T_VertexB_VllVpp` of the `BB_dec` program of the SIMUB package [2].

The first method is based on filling of the single-dimensional array `f4_Integ[n4_cells]` of real numbers which represents numerically the four-dimensional distribution function $f_0(\Theta_{l+}, \Theta_{K+}, \chi; t)$. The array `f4_Integ` contains `n4_cells` $= N(\cos\Theta_{l+}) \times N(\cos\Theta_{K+}) \times N(\chi) \times N(t)$ elements, where $N(\cos\Theta_{l+})$, $N(\cos\Theta_{K+})$, $N(\chi)$ and $N(t)$ are the numbers of bins (generator resolutions) for corresponding variables randomly generated in a four-dimensional volume $V \in \{\cos\Theta_{l+}, \cos\Theta_{K+}, \chi, t\}$. A fast generation of these variables is performed by the function `GetRandom4` by using the algorithm which is analogous to that was realized in the ROOT class `TF3` [19] for generation of three random numbers distributed according to a three-dimensional probability function. In this case the array `f4_Integ` is filled in constructor of the class `T_VertexB_VllVpp` where a large memory space for the array is reserved during all time of existence of this class sample.

The second method involves sequential generation of random numbers according to approach based on usage of the single-dimensional distribution functions given by Eqs. (33) and (34). The corresponding algorithm of generation of the variables t, χ, $\cos\Theta_{l+}$ and $\cos\Theta_{K+}$ is described above. In this case the constructor `T_VertexB_VllVpp` does not fill any arrays and, therefore, it does not reserve a large memory space.

Both methods demonstrated the identical results with reasonable CPU time and memory used.

The B-decay dynamics described by angular correlations (9) should be included without fail into Monte Carlo generators developed for physics studies. The main motivation for developing the new dedicated package SIMUB was that already existing

generators do not take into account all theoretical refinements which are of a great importance for Monte Carlo studies of B-decay dynamics. In particular, in the generators PYTHIA [20] and QQ [21] the time-dependent spin-angular correlations between the final-state particles in the decays $B_s^0(t), \overline{B}_s^0(t) \rightarrow J/\psi(\rightarrow l^+l^-)\,\phi(\rightarrow K^+K^-)$ are not reproduced in a proper way[1].

The time and angular distributions in the helicity frame for decay $B_s^0(t), \overline{B}_s^0(t) \rightarrow J/\psi(\rightarrow \mu^+\mu^-)\,\phi(\rightarrow K^+K^-)$, generated by SIMUB, PYTHIA and QQ packages, are compared in Fig. 3. In case of PYTHIA usage, Fig. 3 shows the uniform distributions for angular variables $\cos\Theta_{K^+}$, $\cos\Theta_{l^+}$ and χ because of lack of time-dependent angular correlations. Due to this simple reason PYTHIA can not be used for Monte Carlo studies of dynamics of sequential two body decays of B mesons in the channels of the type $B \rightarrow V_1(\rightarrow \mu^+\mu^-)\,V_2(\rightarrow P^+P^-)$ with intermediate vector mesons V_1 and V_2. Unfortunately, another well known package QQ turns out to be also not suitable for study of decays of this type because of lack of azimuthal-angle χ correlations.

6   Monte Carlo studies

For Monte Carlo studies of the estimation of physical parameters by applying the angular-moments method, untagged samples of events of the decays $B_s^0(t), \overline{B}_s^0(t) \rightarrow J/\psi\,\phi$ have been generated by using the package SIMUB with various sets of the input values of initial amplitudes $|A_0(0)|$ and $|A_\perp(0)|$ and $\Delta\Gamma_s$. Other parameters are fixed as follows:

$$\delta_1 = \pi\,,\ \delta_2 = 0\,, \Gamma_s = 1/\tau_s = 2.278\ [\text{mm/c}]^{-1}\,,\ \phi_c^{(s)} = 0.04\,.$$

The value of Γ_s used corresponds to the lifetime $\tau_s = 1.464$ ps [12] while the CP-violating weak phase $\phi_c^{(s)}$ was fixed as the upper limit of the constrain (8). The value of $\Delta\Gamma_s$ is expected to be negative in the SM. The combined experimental result

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[1]In the latest version of the generator EvtGen [22] this mode can be also simulated including the full time-dependent spin-angular correlations.

for $|\Delta\Gamma_s|/\Gamma_s$ is not precise: $|\Delta\Gamma_s|/\Gamma_s < 0.52$ at 95% CL [12]. In the approximation of the equal B_s^0 and B_d^0 lifetimes, the $|\Delta\Gamma_s|$ extraction can be improved [12]: $|\Delta\Gamma_s|/\Gamma_s < 0.31$ at 95% CL. A set of the untagged-event samples has been generated with $\Delta\Gamma_s/\Gamma_s \in [-0.3, -0.01]$ to study the influence of $\Delta\Gamma_s$ value on the estimation of $B_s^0(t) \to J/\psi\,\phi$ decay parameters from data analysis.

The values of the time integrated observables $\tilde{b}_i^{(exp)}(T_0)$, defined by Eq. (20), can be extracted from data according to Eq. (22) by summation of weighting functions for each event. The statistical error of $\tilde{b}_i(T_0)$ is defined by

$$(\delta\tilde{b}_i)^{(stat)} = \frac{1}{N(T)}\sqrt{\sum_{j=1}^{N(T_0)}(\tilde{b}_i^{(exp)} - w_i^j)^2}\,, \tag{35}$$

while a systematic error due to limited precision of angular measurements can be estimated as

$$(\delta\tilde{b}_i)^{(sys)} = \sqrt{\frac{\sum_{j=1}^{N(T_0)}\Delta_i^j}{N(T)}}\,. \tag{36}$$

Here

$$\Delta_i^j = \left[\frac{\partial w_i^j}{\partial\cos\Theta_{l+}}\Delta(\cos\Theta_{l+})\right]^2 + \left[\frac{\partial w_i^j}{\partial\cos\Theta_{K+}}\Delta(\cos\Theta_{K+})\right]^2 + \left[\frac{\partial w_i^j}{\partial\chi}\Delta(\chi)\right]^2.$$

In a similar way, the values of the observables $\hat{b}_i^{(exp)}(T_0)$, defined by Eq. (28), can be extracted from the data according to Eq. (29). The formulae for statistical and systematic errors for $\hat{b}_i^{(exp)}(T_0)$ can be obtained by replacement of w_i^j to W_i^j in Eq. (35) and the following redefinition of Δ_i^j in Eq. (36):

$$\Delta_i^j = \left[\frac{\partial W_i^j}{\partial\cos\Theta_{l+}}\Delta(\cos\Theta_{l+})\right]^2 + \left[\frac{\partial W_i^j}{\partial\cos\Theta_{K+}}\Delta(\cos\Theta_{K+})\right]^2 + \left[\frac{\partial W_i^j}{\partial\chi}\Delta(\chi)\right]^2 + \left[\frac{\partial W_i^j}{\partial t}\Delta(t)\right]^2.$$

Eq. (36) can be applied to estimate the systematic errors related both to the measurement precision of the detector and to the limited resolution of the Monte Carlo generator. In the SIMUB generator, for each variable $V \in \{\cos\Theta_{l+}, \cos\Theta_{K+}, \chi, t\}$ randomly generated for decays $B_s^0(t) \to J/\psi(\to l^+l^-)\,\phi(\to K^+K^-)$, the number of bins in the region $[V_{min}, V_{max}]$ was set as $N^{gen} = 50\,000$. The generation precision for the variable V is defined as $\Delta(V) = (V_{max} - V_{min})/N^{gen}$ and systematic errors (36)

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are proportional to $(N^{gen})^{-1/2}$. The B-meson proper time was generated within the interval $t \in [0, T = 2\,\text{mm/c}]$ which includes 99.3% of all B-decays. We have used samples with a maximum of $100\,000$ events of the decay $B_s^0 \to J/\psi\,\phi$ because a statistics of about $80\,000$ events is expected to be obtained per year at the CMS detector at the LHC low luminosity under realistic triggering conditions [23].

Table 2 shows the values of the observables $\tilde{b}_i^{(exp)}(T) \equiv \tilde{b}_i^{(exp)}(T_0 = T)$ extracted from the Monte Carlo data by applying the sets A and B of weighting functions, given by Eqs. (14) and (17), respectively. Various theoretical models for estimation of the transversity amplitudes $|A_0(0)|$ and $|A_\perp(0)|$ (see Table 1) have been considered to fix these parameters in the SIMUB generator. It can be seen from Table 2 that the choice of $N^{gen} = 50\,000$ provides negligibly small systematic errors for the observables as compared with the statistical ones. Moreover, both errors slightly depend on the values of the observables. For observables obtained by using the set-B weighting functions, the statistical errors are significantly smaller than in case of the set-A weighting functions. We should also note that even with the statistics of $100\,000$ events, the values of observables $\tilde{b}_{4,6}^{(exp)}(T)$ and – as consequence of Eq. (27) – the combination $\cos\delta_{1,2}\sin\phi$ cannot be extracted from the data if the CP-violating weak phase $\phi_c^{(s)}$ is small according to the SM expectation (8). In this case, these parameters can be estimated only by using a statistics which is not less than 3×10^9 $B_s^0(t) \to J/\psi\,\phi$ decays.

Analysis of the same Monte Carlo data leads to similar conclusions concerning the behavior of statistical and systematic errors for the observables $\hat{b}_i^{(exp)}(T) \equiv \hat{b}_i^{(exp)}(T_0 = T)$. To illustrate the performance of our method in this case, only the results obtained for transversity amplitudes, corresponding to the Cheng's model [15], are shown in Table 3.

Fig. 4 shows the dependence of the observables $\tilde{b}_i(T)$ and

$$\hat{b}_i'(T) \equiv \frac{1 - e^{-\Gamma_s T}}{\Gamma_s T}\hat{b}_i(T) \quad (i = 1, 2, 3)$$

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on the value of the ratio $\Delta\Gamma_s/\Gamma_s$. For $\Delta\Gamma_s = 0$ we have

$$\tilde{b}_{1,2,3}(T)|_{\Delta\Gamma_s=0} = \hat{b}'_{1,2,3}(T)|_{\Delta\Gamma_s=0} = |A_{0,\|,\perp}(0)|^2 \,.$$

The observables $\tilde{b}_i(T)$ slightly depend on $\Delta\Gamma_s$. The rather strong dependence of the observables $\hat{b}_i(T)$ on the decay width difference $\Delta\Gamma_s$, shown in Fig. 4, can be used for extraction of this parameter from the data analysis as it will be discussed below.

Under the assumption $\phi_c^{(s)} = 0$, we have from Eq. (30):

$$\hat{G}^{(0)}_{L/H}(T) = \pm 2 \frac{e^{\pm\Delta\Gamma_{L/H}T/2} - 1}{\Delta\Gamma_{L/H}} \,.$$

Therefore, the values of the auxiliary parameters $\Delta\Gamma_{L/H}$, defined by Eq. (31), can be determined separately by using the ratios of observables $\hat{b}_i^{(exp)}(T)/\hat{b}_i^{(exp)}(T_0)$, extracted from the data analysis, and solving numerically the equations which arise from one of the following relations:

$$\hat{b}_i(T)/\hat{b}_i(T_0) = \hat{G}_L(T)/\hat{G}_L(T_0) \qquad (i = 1, 2, 5) \tag{37}$$

– to determine $\Delta\Gamma_L$, and the relation

$$\hat{b}_3(T)/\hat{b}_3(T_0) = \hat{G}_H(T)/\hat{G}_H(T_0) \tag{38}$$

– to determine $\Delta\Gamma_H$. Then, the decay-width parameters Γ_s, $\Delta\Gamma_s$ and $\Gamma_{L/H}$ can be determined via Γ' and $\Delta\Gamma_{L/H}$ as

$$\Gamma_s = \Gamma' - \frac{\Delta\Gamma_L - \Delta\Gamma_H}{4}, \qquad \Delta\Gamma_s = \frac{\Delta\Gamma_L + \Delta\Gamma_H}{2}, \qquad \Gamma_{L/H} = \Gamma' \mp \frac{\Delta\Gamma_L}{2}. \tag{39}$$

So, by using some reasonable approximation for Γ' as a starting point for the data analysis, the experimental value of Γ_s can be essentially improved simultaneously with determination of $\Delta\Gamma_s$. The statistical error of Γ_s determination is expected to be twice smaller than for $\Delta\Gamma_s$ determination.

The direct numerical calculations have shown that the difference between the values of observables $\hat{b}_i(T)$ ($i = 1, 2, 3, 5$), calculated with $\phi_c^{(s)} = 0$ and $\phi_c^{(s)} = 0.04$, does not exceed 0.01%. Even in case of statistics of 100 000 events this difference is

negligibly small as compared with statistical errors for these observables (see Table 3). Therefore, the assumption $\phi_c^{(s)} = 0$ is a good approximation for Γ_s, $\Delta\Gamma_s$ and $\Gamma_{L/H}$ determination by the method described above.

Table 4 shows the results of determination of the decay-width parameters after applying the described procedure to the Monte Carlo data. The sample of 100 000 events generated in case of Cheng's model with $\Delta\Gamma_s/\Gamma_s = -0.15$ has been used. Both sets A and B of weighting functions have been applied to extract the observables $\hat{b}_i^{(exp)}$. The value of Γ', which is treated as some arbitrary initial approximation for the total decay width of B_s^0-meson, was fixed as $\Gamma' = 1.05\,\Gamma_s$, i.e. it was shifted by 5% relative to the "true" value of Γ_s fixed in the Monte Carlo generator SIMUB. The value of $T_0 = 0.1\,T$ was chosen as it provides the minimal statistical errors to determine the ratios $\hat{b}_i^{(exp)}(T)/\hat{b}_i^{(exp)}(T_0)$. In Table 4 we present the result for $\Delta\Gamma_L$ obtained from the ratio $\hat{b}_1^{(exp)}(T)/\hat{b}_1^{(exp)}(T_0)$ only, which gives the best precision. Table 4 shows that the set-B weighting functions give more precise and stable results than the set-A functions.

To improve the precision of $\Delta\Gamma_s$ determination, the same procedure should be repeated with Γ' fixed to be equal to the value of Γ_s determined at the first step. Because of $\Delta\Gamma_s = \Delta\Gamma_L = \Delta\Gamma_H$ in case of $\Gamma' = \Gamma_s$, the value of $\Delta\Gamma_s$ is defined at the second step to be equal to the value of $\Delta\Gamma_L$ determined from the ratio $\hat{b}_1^{(exp)}(T)/\hat{b}_1^{(exp)}(T_0)$ using Eq. (37). Using the values of $\Delta\Gamma_s$ from Table 4 as an input value of Γ' at the second step, we have obtained finally the following results (to be compared with the input value $\Delta\Gamma_s = -0.3418$ set in the SIMUB generator):

$$\Delta\Gamma_s^{exp} = -0.330 \pm 0.057 \pm 0.004 \quad \text{(set A)},$$
$$\Delta\Gamma_s^{exp} = -0.338 \pm 0.034 \pm 0.002 \quad \text{(set B)}.$$

This way one can reduce not only the statistical error but also essentially improve the stability of the $\Delta\Gamma_s$ result even in case of using the set-A weighting functions.

Table 5 shows the statistical errors of $\Delta\Gamma_s/\Gamma_s$ determination by the described approach applied to different statistics of Monte Carlo events generated with various

"true" values of $\Delta\Gamma_s$. The lack of numbers in the table corresponds to cases when the approach is not able to give a certain result for $\Delta\Gamma_s$. The use of set-B weighting functions gives more stable results even in case of too small statistics and values of $\Delta\Gamma_s$, for which the same approach does not work with set-A functions. The sensitivity of the method is measured by the statistical error of $\Delta\Gamma_s/\Gamma_s$, which only slightly depends on the value of this ratio and is proportional to $1/\sqrt{N}$, where N is the number of events. In particular, for a statistics $100\,000$ events, the statistical error is about 0.015, while for 1000 events – about 0.15.

The value of $\Delta\Gamma_s$ can be determined similarly by using the ratios $\tilde{b}_i^{(exp)}(T)/\tilde{b}_i^{(exp)}(T_0)$ or $\hat{b}_i^{(exp)}(T)/\tilde{b}_i^{(exp)}(T)$, extracted from the data analysis with $\Gamma' = \Gamma_s$, and solving the equations arising from the relations

$$\tilde{b}_i(T)/\tilde{b}_i(T_0) = \tilde{G}_L(T)/\tilde{G}_L(T_0) \quad (i = 1, 2, 5)\,, \qquad \tilde{b}_3(T)/\tilde{b}_3(T_0) = \tilde{G}_H(T)/\tilde{G}_H(T_0)$$

or

$$\hat{b}_i(T)/\tilde{b}_i(T) = \hat{G}_L(T)/\tilde{G}_L(T) \quad (i = 1, 2, 5)\,, \qquad \hat{b}_3(T)/\tilde{b}_3(T) = \hat{G}_H(T)/\tilde{G}_H(T)\,.$$

But in both these cases the precision of $\Delta\Gamma_s$ determination turns out to be worse than in the approach based on the ratios $\hat{b}_i^{(exp)}(T)/\hat{b}_i^{(exp)}(T_0)$ because of the weak $\Delta\Gamma_s$-dependence of the $\tilde{b}_i(T)$ observables.

The initial transversity amplitudes and strong-phase difference can be recalculated from the values of observables $\tilde{b}_i^{(exp)}(T)$ according to Eq. (26). The results of such determination of the parameters $|A_f(0)|^2$ ($f = 0, ||, \perp$) and $\cos(\delta_2 - \delta_1)$ are shown in Table 6 for different statistics. We have used the Monte Carlo sample generated with the theoretical values of the amplitudes $|A_0(0)|$ and $|A_\perp(0)|$ corresponding to Cheng's model [15]. To extract the observables $\tilde{b}_i^{(exp)}(T)$, the set B of the weighting function has been applied to Monte Carlo data. To estimate the statistical errors for parameters $|A_f(0)|^2$ ($f = 0, ||, \perp$) and $\cos(\delta_2 - \delta_1)$, the standard error-propagation method has been applied to the statistical errors of the observables $\tilde{b}_i^{(exp)}(T)$, taking into account the correlation between pairs of different observables. The systematic

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errors of the observables related to the limited generator resolution are neglected. The total errors for parameters $|A_f(0)|^2$ ($f = 0, ||, \perp$) should also include the additional uncertainty related to the error of calculation of $\tilde{\gamma}(T)$ caused by the error of $\Delta\Gamma_s$ (see definition of $\tilde{\gamma}(T)$ in Eq. (25) and Eq. (23)). In Table 6 we also show these errors calculated by assuming $\Delta\Gamma_s = -0.15\,\Gamma_s$ (see Table 5)

$$
\frac{\delta(\Delta\Gamma_s)}{|\Delta\Gamma_s|} = \begin{cases} 30\% & \text{for } 10\,000 \text{ events,} \\ 9.3\% & \text{for } 100\,000 \text{ events.} \end{cases} \tag{40}
$$

7   Conclusion

For the decay $B_s^0 \to J/\psi\,\phi$ in the framework of the method of angular moments a non-fit scheme for separate estimation of the parameters $\Delta\Gamma_s$, Γ_s and $|A_f(0)|^2$ ($f = 0, ||, \perp$) has been proposed, based on analysis of an untagged sample, and studied by Monte Carlo method. A strong dependence of statistical measurement errors on the choice of the weighting functions has been demonstrated. The statistical error of the ratio $\Delta\Gamma_s/\Gamma_s$ for values in the interval $[0.03, 0.3]$ was found to be independent of the central value and amounts to about 0.015 for 10^5 events. The method of angular moments gives stable results for the estimate of $\Delta\Gamma_s$ and is found to be an efficient and flexible tool for the quantitative investigation of the $B_s^0 \to J/\psi\,\phi$ decay.

Acknowledgements

A   General information on the SIMUB package

The generator SIMUB was developed for Monte Carlo simulation of B-meson production and decays at the CMS detector. The SIMUB package provides two regimes of B-decay generation:

- simulation of decay after fragmentation according to angular distribution and time dependence (in case of a neutral B-meson) governed by spin-angular correlations, time evolution and other theoretical refinements – *dynamical mode*,

- simulation of decay by PYTHIA into phase space – *PYTHIA kinematical mode*.

The package is kept under the directory `SIMUB` which has the following structure:

- `bb_gen` – routines needed to generate events with $b\bar{b}$-pairs at parton level (FORTRAN, PYTHIA, HBOOK);

- `bb_frg` – routines performing string fragmentation, generation of B-mesons, and decays of particles in the PYTHIA kinematical mode (FORTRAN, PYTHIA, HBOOK); storing the results into standard HEPEVT or PYJETS Ntuples for further usage in the CMS simulation or for analysis;

- `BB_dec` – routines performing B-decays in dynamical mode (C++, ROOT);

- `include` – a collection of common blocks for the routines `bb_gen`, `bb_frg` and `BB_dec`;

- `lib` – the PYTHIA source codes and object files;

- `doc` – documentation.

The data flow between the three main parts (`bb_gen`, `bb_frg`, and `BB_dec`) of the package SIMUB is shown in Fig. 5 and organized as follows:

- At the first stage the events containing $b\bar{b}$-pairs are generated by the program `bb_gen`, stopped before fragmentation and written to an intermediate Ntuple to store events at parton level.

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- At the next step the program `bb_frg` reads $b\bar{b}$-events from the intermediate Ntuple and performs the string fragmentation and the decays of all particles in the PYTHIA kinematical mode with exception of B-mesons selected according to user directives to be decayed in dynamical mode.

- Then, the information about the selected B-mesons is used by the program `BB_dec` to perform their decays according to dynamical modes.

- Finally, the information about the selected B-mesons and products of their decays is transfered to the subprogram `bb_frg` to be added to the information about other particles and stored in a final output Ntuple.

The package SIMUB is installed and tested on Linux (RedHat 6.x, 7.x) platforms. The package and documentation for user is found on the SIMUB Package Home Page:

http://cmsdoc.cern.ch/~shulga/SIMUB/SIMUB.html.

To install the program, the file `SIMUB.tar.gz` should be copied from the SIMUB Package Home Page to the user directory and unpacked to obtain the directory `SIMUB`.

The current version of the program SIMUB is adopted to the usage of the generator PYTHIA [20] (version 6.215), CERNLIB-2002 [24], and ROOT package [19] (version 3.05/07). The standard CERN Program Library should be installed at a Linux machine according to the record in the files `bb_gen/mak/Makefile` and `bb_frg/mak/Makefile`:

`CERNLIB := 'cernlib pawlib graflib mathlib packlib kernlib'`

or the record in the file `BB_dec/mak/Makefile`:

`SYSLIBS = -L/cer/new/lib/ -lpacklib ...`

The user has to generate the object file of the PYTHIA generator in the directory `lib` and to install the ROOT package. Detailed instructions on the installation,

compilation and running of the package SIMUB are given in the documents "User and Developer Guide" and "Quick Start". The postscript files of these documents are placed in the directory `SIMUB/doc` and also found on the SIMUB Package Home Page.

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Table 1: Predictions for $B_s^0 \to J/\psi\,\phi$ (in brackets – for $B_d^0 \to J/\psi\,K^\star$) observables obtained in Ref. [1] for various model estimates of the $B \to K^\star$ form factors [13, 14, 15] (the normalization condition $|A_0(0)|^2 + |A_{||}(0)|^2 + |A_\perp(0)|^2 = 1$ is implied)

Observable	BSW [13]	Soares [14]	Cheng [15]
$|A_0(0)|^2$	0.55 (0.57)	0.41 (0.42)	0.54 (0.56)
$|A_\perp(0)|^2$	0.09 (0.09)	0.32 (0.33)	0.16 (0.16)

Table 2: Comparison of the observables $\tilde{b}_i^{(exp)}(T)$, extracted from Monte Carlo data, with their values $\tilde{b}_i^{(th)}(T)$ corresponding to various theoretical models for $|A_0(0)|$ and $|A_\perp(0)|$. A sample of $100\,000$ decay events generated with $\Delta\Gamma_s/\Gamma_s = -0.15$ was used. The first errors are statistical (see Eq. (35)) while the second errors correspond to the systematic uncertainties only from limited angular precision (see Eq. (36))

a) BSW model [13]:

i	$\tilde{b}_i^{(th)}(T)$	$\tilde{b}_i^{(exp)}(T)$ (set A)	$\tilde{b}_i^{(exp)}(T)$ (set B)
1	0.5425	$0.5409 \pm 0.0044 \pm 0.0003$	$0.5432 \pm 0.0024 \pm 0.0002$
2	0.3551	$0.3619 \pm 0.0047 \pm 0.0004$	$0.3579 \pm 0.0036 \pm 0.0004$
3	0.1024	$0.0972 \pm 0.0049 \pm 0.0004$	$0.0991 \pm 0.0034 \pm 0.0004$
4	-0.00055	$-0.0017 \pm 0.0037 \pm 0.0004$	$-0.0021 \pm 0.0033 \pm 0.0003$
5	-0.4389	$-0.4344 \pm 0.0050 \pm 0.0003$	$-0.4344 \pm 0.0050 \pm 0.0003$
6	0.00067	$0.0037 \pm 0.0055 \pm 0.0003$	$0.0037 \pm 0.0055 \pm 0.0003$

b) Model by Soares [14]:

i	$\tilde{b}_i^{(th)}(T)$	$\tilde{b}_i^{(exp)}(T)$ (set A)	$\tilde{b}_i^{(exp)}(T)$ (set B)
1	0.3908	$0.3900 \pm 0.0046 \pm 0.0003$	$0.3955 \pm 0.0023 \pm 0.0002$
2	0.2574	$0.2617 \pm 0.0049 \pm 0.0004$	$0.2551 \pm 0.0037 \pm 0.0004$
3	0.3518	$0.3483 \pm 0.0047 \pm 0.0004$	$0.3509 \pm 0.0037 \pm 0.0004$
4	-0.00086	$-0.0083 \pm 0.0040 \pm 0.0004$	$-0.0017 \pm 0.0035 \pm 0.0003$
5	-0.3171	$-0.3156 \pm 0.0052 \pm 0.0003$	$-0.3156 \pm 0.0052 \pm 0.0003$
6	0.0011	$0.0008 \pm 0.0052 \pm 0.0003$	$0.0008 \pm 0.0052 \pm 0.0003$

c) Model by Cheng [15]:

i	$\tilde{b}_i^{(th)}(T)$	$\tilde{b}_i^{(exp)}(T)$ (set A)	$\tilde{b}_i^{(exp)}(T)$ (set B)
1	0.5271	$0.5228 \pm 0.0045 \pm 0.0003$	$0.5267 \pm 0.0024 \pm 0.0001$
2	0.2928	$0.2980 \pm 0.0048 \pm 0.0004$	$0.2950 \pm 0.0036 \pm 0.0004$
3	0.1801	$0.1791 \pm 0.0048 \pm 0.0004$	$0.1778 \pm 0.0035 \pm 0.0004$
4	-0.00066	$-0.0030 \pm 0.0037 \pm 0.0003$	$-0.0034 \pm 0.0034 \pm 0.0003$
5	-0.3928	$-0.3927 \pm 0.0051 \pm 0.0003$	$-0.3927 \pm 0.0051 \pm 0.0003$
6	0.00088	$-0.0019 \pm 0.0054 \pm 0.0003$	$-0.0019 \pm 0.0054 \pm 0.0003$

Table 3: Comparison of the values of observables $\hat{b}_i^{(exp)}(T)$, with $\Gamma' = \Gamma_s$, extracted from the Monte Carlo data, with their values $\hat{b}_i^{(th)}(T)$ corresponding to the model of Cheng [15] for initial transversity amplitudes

i	$\hat{b}_i^{(th)}(T)$	$\hat{b}_i^{(exp)}(T)$ (set A)	$\hat{b}_i^{(exp)}(T)$ (set B)
1	2.2036	$2.176 \pm 0.044 \pm 0.003$	$2.206 \pm 0.026 \pm 0.001$
2	1.2242	$1.282 \pm 0.045 \pm 0.004$	$1.245 \pm 0.034 \pm 0.004$
3	0.9187	$0.917 \pm 0.045 \pm 0.004$	$0.930 \pm 0.034 \pm 0.004$
4	-0.0073	$-0.099 \pm 0.036 \pm 0.003$	$-0.094 \pm 0.032 \pm 0.003$
5	-1.6425	$-1.618 \pm 0.048 \pm 0.003$	$-1.618 \pm 0.048 \pm 0.003$
6	0.0098	$0.067 \pm 0.050 \pm 0.003$	$0.067 \pm 0.050 \pm 0.003$

Table 4: Results of determination of the decay-width parameters (in units $(\mathrm{mm}/c)^{-1}$) based on extraction of the observables $\hat{b}_i^{(exp)}$ from analysis of $100\,000$ Monte Carlo events. The input value of $\Delta\Gamma_s$ corresponds to $\Delta\Gamma_s/\Gamma_s = -0.15$

Parameter	Input value	Measurement (set A)	Measurement (set B)
$\Delta\Gamma_L$	-0.1139	$-0.103 \pm 0.058 \pm 0.003$	$-0.110 \pm 0.034 \pm 0.002$
$\Delta\Gamma_H$	-0.5696	$-0.478 \pm 0.137 \pm 0.012$	$-0.554 \pm 0.101 \pm 0.012$
Γ_L	2.4493	$2.444 \pm 0.029 \pm 0.002$	$2.447 \pm 0.017 \pm 0.001$
Γ_H	2.1076	$2.154 \pm 0.068 \pm 0.006$	$2.115 \pm 0.050 \pm 0.006$
Γ_s	2.2784	$2.299 \pm 0.037 \pm 0.003$	$2.281 \pm 0.027 \pm 0.003$
$\Delta\Gamma_s$	-0.3418	$-0.290 \pm 0.074 \pm 0.006$	$-0.332 \pm 0.053 \pm 0.006$

Table 5: Statistical errors of $\Delta\Gamma_s$ extraction (in units $[\text{mm/c}]^{-1}$) obtained by applying the angular-moments method with set-B (set-A) weighting functions to the Monte Carlo data samples with different numbers of events

$\Delta\Gamma_s/\Gamma_s$	200 events	500 events	10^3 events	10^4 events	10^5 events
-0.03	-	-	-	0.035 (-)	0.014 (0.023)
-0.05	-	-	-	0.046 (-)	0.014 (0.022)
-0.1	-	-	0.11 (-)	0.046 (0.079)	0.014 (0.024)
-0.15	-	-	0.13 (0.19)	0.045 (0.078)	0.014 (0.024)
-0.2	-	0.23 (-)	0.12 (0.18)	0.048 (0.072)	0.015 (0.026)
-0.3	0.21 (-)	0.23 (-)	0.18 (0.20)	0.050 (0.083)	0.016 (0.028)

Table 6: Determination of initial transversity amplitudes and strong-phase difference by using the values of observables $\tilde{b}_i^{(exp)}(T)$ extracted from Monte Carlo data. The events sample has been generated for the case of Cheng's model [15] for transversity amplitudes and with $\Delta\Gamma_s = 0.15\,\Gamma_s$. The first errors are statistical while the second errors are caused by uncertainties of $\Delta\Gamma_s$ determination

Parameter	Input value	10 000 events	100 000 events
$|A_0(0)|^2$	0.54	$0.527 \pm 0.007 \pm 0.012$	$0.5398 \pm 0.0023 \pm 0.0011$
$|A_{||}|^2$	0.30	$0.337 \pm 0.011 \pm 0.008$	$0.3023 \pm 0.0036 \pm 0.0006$
$|A_\perp|^2$	0.16	$0.136 \pm 0.010 \pm 0.020$	$0.1579 \pm 0.0032 \pm 0.0018$
$\cos(\delta_2 - \delta_1)$	-1	-1.021 ± 0.044	-0.9962 ± 0.015

Figure captions

Figure 1. Definition of physical angles for description of decays $B_s^0(t), \overline{B}_s^0(t) \to J/\psi(\to l^+l^-)\,\phi(\to K^+K^-)$ in the helicity frame.

Figure 2. Color suppressed diagrams for decays $B_q^0 \to J/\psi V$ $((q,V) \in \{(s,\phi),(d,K^\star)\})$.

Figure 3. Comparison of time and angular distributions for the decays $B_s^0(t), \overline{B}_s^0(t) \to J/\psi(\to \mu^+\mu^-)\,\phi(\to K^+K^-)$ generated by SIMUB, PYTHIA and QQ packages. For time t we use units $1\mathrm{mm}/c \approx 3.33 \times 10^{-12}$ sec.

Figure 4. Dependence of the observables $\tilde{b}_i(T)$ and $\hat{b}'_i(T) \equiv \hat{b}_i(T)[1-\exp(-\Gamma T)]/(\Gamma T)$ $(i = 1, 2, 3)$ on the value of $\Delta\Gamma_s/\Gamma_s$. The observables have been calculated for the case of Cheng's model for transversity amplitudes.

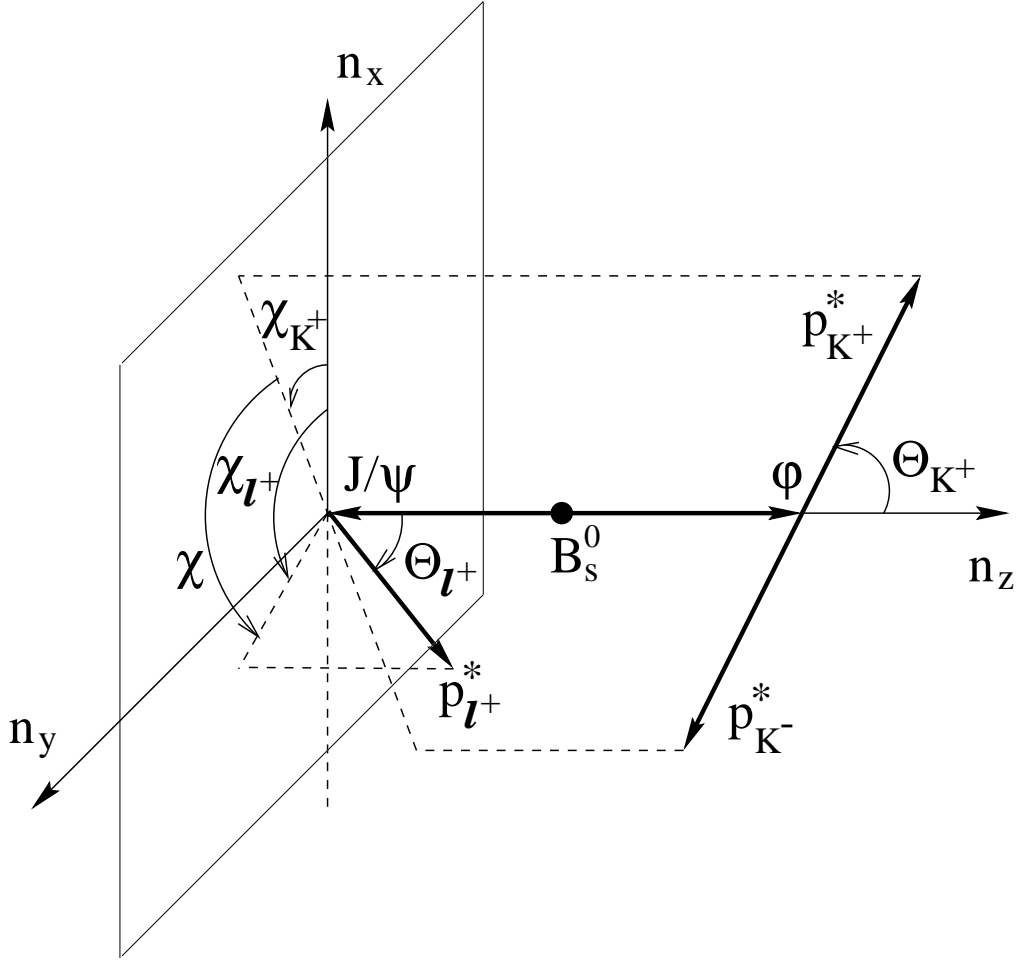**Figure 5.** The flow of data within the package SIMUB.
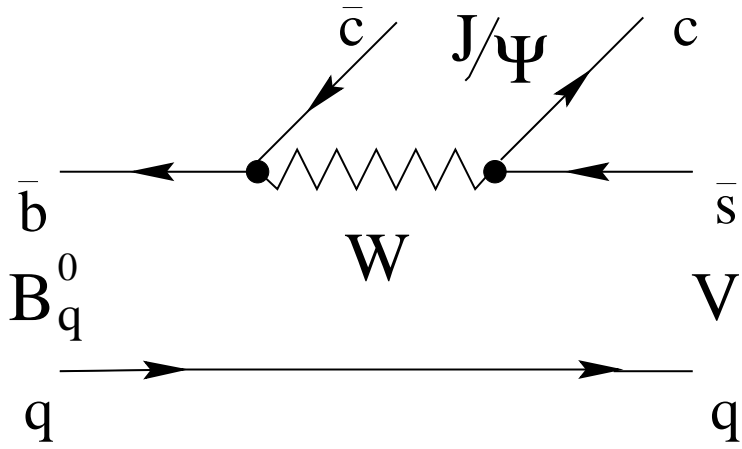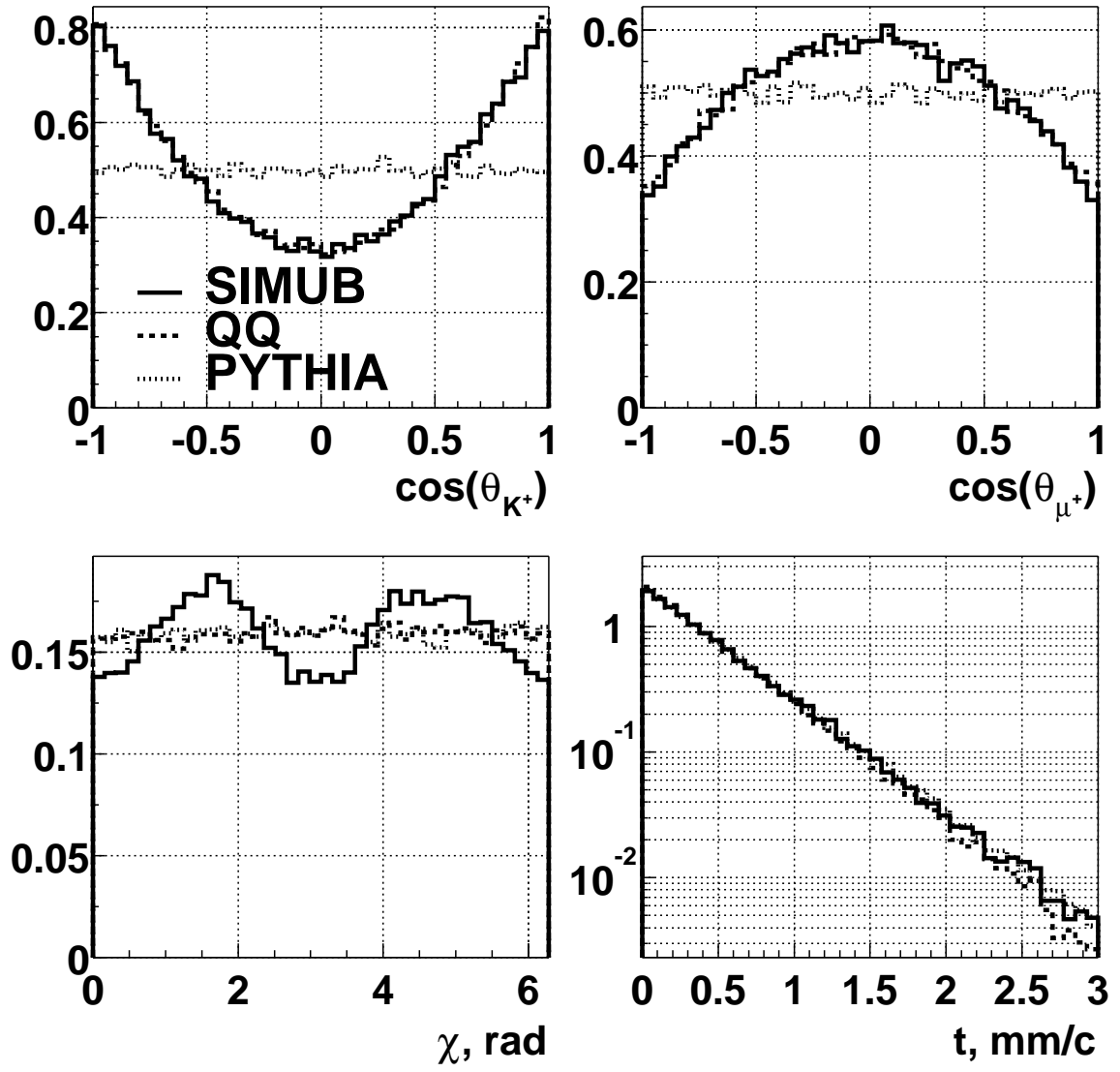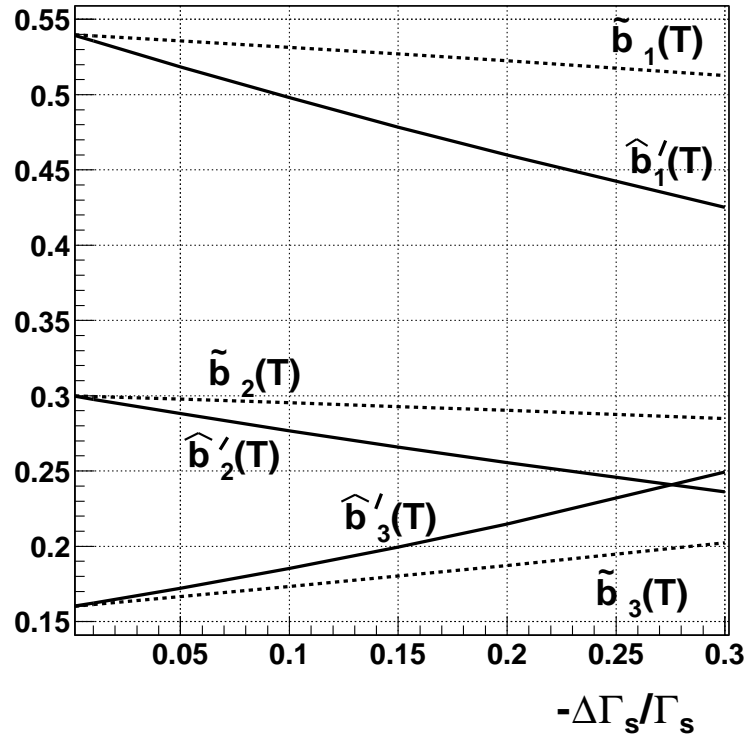
Figure 1

Figure 2

Figure 3

$\tilde{b}_1(T)$

$\widehat{b}'_1(T)$

$\tilde{b}_2(T)$

$\widehat{b}'_2(T)$

$\widehat{b}'_3(T)$

$\tilde{b}_3(T)$

$-\Delta\Gamma_s/\Gamma_s$

0.55
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15

0.05   0.1   0.15   0.2   0.25   0.3

Figure 4

bb_gen

User directives

FORTRAN

PYTHIA

HBOOK

bb_frg

User directives

FORTRAN

PYTHIA

HBOOK

BB_dec

CERNLIB, g2c modul

C++

ROOT

Final Ntuple
in HEPEVT or PYJETS formats

Intermediate Ntuple with events
at parton level in PYJETS format

Figure 5